Appendix

The structure of the model is represented by the following set of differential equations.

\[
\begin{align*}
\frac{dX_{k_1}^{00}}{dt} & = \Lambda_{k_1}(t) \sigma_{k_1}X_{k_1}^{01}(t) (\mu + \lambda_{k_1}(t) + \rho_{k_1}(t)) X_{k_1}^{00}(t) + \gamma_{k_1}^{01}(T2) + R_{k_1}^{00}(t), \\
\frac{dX_{k_1}^{10}}{dt} & = \lambda_{k_1}(t)X_{k_1}^{00}(t) \sigma_{k_1}X_{k_1}^{11}(t) (\mu + 1 + \rho_{k_1}(t)) X_{k_1}^{10}(t) + C_{k_1}^{11}(T2) + R_{k_1}^{10}(t)(0.2), \\
\frac{dX_{k_1}^{20}}{dt} & = \gamma_{k_1}X_{k_1}^{10}(t) \sigma_{k_1}X_{k_1}^{21}(t) (\mu + \gamma_{k_1}(t)) X_{k_1}^{20}(t) + C_{k_1}^{21}(T2), \\
\frac{dX_{k_1}^{30}}{dt} & = \gamma_{2}X_{k_1}^{20}(t) \sigma_{k_1}X_{k_1}^{31}(t) (\mu + \rho_{k_1}(t)) X_{k_1}^{30}(t) + C_{k_1}^{31}(T2), \\
\frac{dX_{k_1}^{01}}{dt} & = \rho_{k_1}(t)X_{k_1}^{00}(t) (\mu + \sigma_{k_1}) X_{k_1}^{01}(t) + C_{k_1}^{01}(T), \\
\frac{dX_{k_1}^{11}}{dt} & = \lambda_{2}X_{k_1}^{01}(t) \rho_{k_1}(t)X_{k_1}^{10}(t) (\mu + \sigma_{k_1}) X_{k_1}^{11}(t) + C_{k_1}^{11}(T), \\
\frac{dX_{k_1}^{21}}{dt} & = \gamma_{1}X_{k_1}^{11}(t) \rho_{k_1}(t)X_{k_1}^{20}(t) (\mu + \sigma_{k_1}) X_{k_1}^{21}(t) + C_{k_1}^{21}(T), \\
\frac{dX_{k_1}^{31}}{dt} & = \gamma_{2}X_{k_1}^{21}(t) \rho_{k_1}(t)X_{k_1}^{30}(t) (\mu + \sigma_{k_1}) X_{k_1}^{31}(t) + C_{k_1}^{31}(T), \\
\frac{dX_{k_1}^{40}}{dt} & = \gamma_{1}X_{k_1}^{30}(t) \sigma_{k_1}X_{k_1}^{41}(t) (\mu + \lambda_{k_1}(t)) X_{k_1}^{40}(t) + C_{k_1}^{40}(T), \\
\frac{dX_{k_1}^{41}}{dt} & = \gamma_{3}X_{k_1}^{31}(t) (\mu + \sigma_{k_1}) X_{k_1}^{41}(t) + C_{k_1}^{41}(T).
\end{align*}
\]

Screening and treatment of GC:

With: \( C_{k_1}^{11}(T2) \) \( C_{k_1} \% \cdot F_{k_1} \cdot X_{k_1}^{11}(t) \cdot \| t \| \theta_1 \), \( \forall h, k \).

\[
\text{for } \theta: \begin{array}{c|c}
  t & \text{if } t \neq T2 \\
\end{array}
\]

Condom use per partnership:

\[
\beta_{\text{new}_{k,i,j}}(t) = (\text{Cond}_{k} \cdot (T2) \% \cdot \{ t \theta_2 \}) \quad 1 + (\text{Cond}_{k,i,j}(T1) \% \cdot \{ t \theta_1 \}) \beta_{k,i,j}, \\
\xi_{\text{new}_{k,i,j}}(T2) = (\text{Cond}_{k,i,j}(T2) \% \cdot \{ t \theta_2 \}) \quad (\text{Cond}_{k,i,j}(T1) \% \cdot \{ t \theta_1 \}) \xi_{k,i,j},
\]

\( \forall h, k \)
with \( \text{Cond}_{k,i,j}(t) \) \( \forall k, i, j \)

\[
\begin{align*}
\text{or } \theta_1 & \quad \begin{cases} 
  t & \text{if } t \neq T_1 \\
  T_1 & \text{if } t \geq T_1 
\end{cases} \\
\text{or } \theta_2 & \quad \begin{cases} 
  t & \text{if } t \neq T_2 \\
  T_2 & \text{if } t \geq T_2 
\end{cases}
\end{align*}
\]

Here \( X_{ki}^{hs}(t) \) represents the number of individuals of sex \( k \) (1: female; 2: male) and activity class \( i \) (1: low rate of partner change; 2: higher rates of partner change) in disease states \( h \) (HIV status) and \( s \) (STD status) at time \( t \). An individual is in state \( h = 0 \) if susceptible to HIV in state \( h = 1 \) if in phase 1 of HIV infection, in state \( h = 2 \), or \( h = 3 \) if in phase 2 and 3 of HIV infection respectively. An individual in state \( h = 4 \) is diagnosed with full blown AIDS. An individual is susceptible to the STD if \( s = 0 \) and currently infected with the STD if \( s = 1 \). \( C_{ki} \) represents the fraction of individuals in risk group \( k, i \) screened/diagnosed and treated at time \( T_2 \) (coverage) \( F_{ki} \) is the frequency of screening in activity class \( i \) of sex \( k \) per year. \( \text{Cond}_{k,i,j}(T_1) \) and \( \text{Cond}_{k,i,j}(T_2) \) are the fraction of individuals \( k, i \) who start using condoms in partnership with someone of opposite sex \( k' \) and activity class \( j \) at time \( T_1 \) and \( T_2 \).

An individual susceptible to both infections \( (X_{ki}^{00}(t)) \) can get infected either by the STD or HIV at a rate \( \rho_{ki}(t) \) (eqn 0.7) and \( \lambda_{1ki}(t) \) (eqn 0.8) respectively. It is assumed that HIV positives and negatives are equally likely to acquire an STD infection. However, HIV negative individuals \( (X_{ki}^{01}(t) \text{ or } X_{ki}^{00}(t)) \) are more likely to get HIV from HIV-STD partners. HIV positive individuals \( (X_{ki}^{01}(t)) \) are more susceptible to infection with HIV \( (\lambda_{2ki}(t)) \) (eqn 0.9). At time \( t = 0 \): e. 1980, \( \sum_{i=1}^{2} X_{ki}^{00}(0) = 122605 \)

20 ems
and $\sum_{i=1}^{2} X_{20}^{10} = 115558$ and the HIV epidemic is seeded with $X_{11}^{10}(0) = 1, X_{12}^{10}(0) = 1$ while the STD is already at its initial equilibrium prevalence before the introduction of HIV. The other state variables are equal to 0. Once infected, an STD positive of sex $k$ and activity class $i$ remains infected and infectious for a period of time of $1/\sigma_{ki}$ respectively after which period, the individual joins the susceptible class again. Whether STD positive or negative, HIV seroconverters remain symptomless for an average period of time $(1/\gamma 1 + 1/\gamma 2 + 1/\gamma 3)$ before progressing to the AIDS stage. Individuals in the population die at a natural rate $\mu$ ($\mu = $average sexual life expectancy). The AIDS related mortality, $\alpha$ is equal to $1/$life expectancy of patient with full blown AIDS.

The per capita rate of STD infection is defined as follows:

$$\rho_{ki}(t) = m_{ki}(t) \sum_{i} \xi_{new_{ki}} \varphi_{ki}(t) \sum_{h=0}^{3} \frac{X_{k,j}^{h1}(t)}{\sum_{s} X_{k,j}^{s}(t)}$$

(0.11)

The per capita rates of HIV infection for those without (STD) and with (STD) current infection with the cofactor STD are defined as follows:

$$\lambda_{1_{ki}}(t) = m_{ki}(t) \sum \varphi_{ki}(t) \frac{\sum_{h=1}^{3} \beta new_{ki}^{h}(X_{k,j}(t)^{h0} + RR_{HIV/STD} \cdot X_{k,j}(t)^{h1}(t))}{\sum_{s} X_{k,j}^{s}(t) \sum_{h} X_{k,j}^{h}(t)}$$

(0.12)

As seen in equations (0.7-0.8), the rates of infection depend on the annual rate of partner acquisition ($m_{ki}(t)$) at time of an individual of sex $k$ and activity level $i$; the probability of choosing an infected partner, which is a function of the mixing pattern $\varphi_{ki}(t)$.
(the probability that a member of class $i$ and sex $k$ has selected a partner of sex $k'$ and class $j$ at time $t$); the probability that this partner is infected (prevalence of infection in risk group $k,i$); and the per partner transmission probability of the STD ($\xi_{new_{k'j}}$) or HIV ($\beta_{new_{j}}$) from a partner of sex $k'$, activity class $j$ to a susceptible of sex $k$ and activity class.

AIDS patients are assumed not to contribute to transmission because of the severity of their illness. The term $RR_{HIV/STD}$ in (0.13) and in the right hand side of (0.12) represents the relative increase in HIV susceptibility of HIV STD and in infectivity of STD+ /HIV+ respectively.

New recruits join the susceptible sexually active population at a rate $\Lambda_{ki}(t)$. The expression for the new recruits to the sexually active population is:

$$
\Lambda_{ki}(t) = Q_{ki}RPf \left[ X_{t1}^{01}(t, \tau) + X_{t1}^{11}(t, \tau) \right]
+ X_{t1}^{11}(t, \tau) + X_{t1}^{21}(t, \tau) + X_{t1}^{21}(t, \tau) \right]
$$

(0.14)

Here $Q_{ki}$ is the initial distribution in sexual activity for each sex in the absence of AIDS induced mortality. $R$ denotes the sex ratio (assumed to be 1:1 at birth which gives $R = 0.5$). $f$ denotes the per capita birth rate of sexually active women $f = 0.3$. $P$ is the proportion of uninfected infants who survive to join the sexually active age classes[49] ($P = \exp^{-br}$ where $\tau$ is the age at sexual maturity (15 years) and $b$ is the death rate over the interval $[0, \tau]$). The net rate of prostitute renewal is given by:

$$
R_{ki}^{00} = \gamma_{3} \cdot X_{ki}^{30}(t)/2, \text{ for } k \neq 1, i \neq 2
$$

(0.15)

$$
R_{ki}^{00} = 0, \text{ otherwise}
$$

(0.16)
We define the elements of the mixing matrix as [49]

\[
\varphi_{kij} = \frac{W_{kij} \left( N_{k_i}(t) N_{l_j}^4(t) \right) m_{k_i}(t) - N_k^4(0)}{\sum_k W_k \left( N_k N_k^4(0) \right)^{1/4}} \quad \text{or} \quad k\neq i
\]

(0.19)

The \( W_k \) define a set of weights which represent the preference of someone of sex \( k \) in activity class \( i \) for someone of the opposite sex in activity class \( j \). The elements of the mixing matrix \( \varphi_{kij}(t) \) must satisfy the following constraint at time \( t \) [49]:

\[
D \varphi_{kij}(t) = \varphi_{kij}(t)
\]

(0.20)

\[
\sum k \varphi_{kij}(t) = N_{k_i}(t) N_{k_j}^4(t) m_{k_i}(t) - N_k^4(t) m_{k_j}(0) - N_k^4(t) m_{k_i}(0)
\]

(0.22)

As described in ref [49] we adopt a procedure which means rate of partner change in the lowest female activity group remains unchanged (for all \( t \)) to act as a reference point to define a rate of change as mortality influences population structure condition. The elements of the mixing matrix should be equal for males and females (details in ref [49]).

For the high prevalence scenario, we used the weights: $W_{111}(1980) = 2$, $W_{112}(1980) = W_{1980} = 1$, $W_{211}(1980) = 2$, $W_{212}(1980) = 1$ and $W_{221}(1980) = W_{222}(1980)$. To obtain the same $Gc$ equilibrium prevalence in 1980 than for Cotonou, the following $Gc$ recovery rates were used: $\sigma_{11} = 6.5, \sigma_{12} = 8.4, \sigma_{21} = 6.5, \sigma_{22} = 8.4$. 

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