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Dynamics of non-cohabiting sex partnering in sub-Saharan Africa: a modelling study with implications for HIV transmission

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ABSTRACT

Objective To develop an analytical understanding of non-cohabiting sex partnering in sub-Saharan Africa (SSA) using nationally representative sexual behaviour data.

Method A non-homogenous Poisson stochastic process model was used to describe the dynamics of non-cohabiting sex. The model was applied to 25 countries in SSA and was fitted to Demographic and Health Survey data. The country-specific mean values and variances of the distributions of number of non-cohabiting partners were estimated.

Results The model yielded overall robust fits to the empirical distributions stratified by marital status and sex. The median across all country-specific mean values was highest for unmarried men at 0.574 non-cohabiting partners over the last 12 months, followed by that of unmarried women at 0.337, married men at 0.192 and married women at 0.038. The median of variances was highest for unmarried men at 0.127, followed by married men at 0.057, unmarried women at 0.003 and married women at 0.000. The largest variability in mean values across countries was for unmarried men (0.103–1.206), and the largest variability in variances was among unmarried women (0.000–1.994).

Conclusions Non-cohabiting sex appears to be a random 'opportunistic' phenomenon linked to situations that may facilitate it. The mean values and variances of number of partners in SSA show wide variation by country, marital status and sex. Unmarried individuals have larger mean values than their married counterparts, and men have larger mean values than women. Unmarried individuals appear to play a disproportionate role in driving heterogeneity in sexual networks and possibly epidemiology of sexually transmitted infections.

INTRODUCTION

The disease burden of sexually transmitted infections (STIs) including HIV is a major public health challenge for developed and developing countries.¹ Since STIs propagate through sexual contact, understanding the dynamics of STI transmission in human populations is predicated on a satisfactory understanding of the patterns of sexual partnering and structure of sexual networks.^{2–4} This understanding however is challenged by the difficulty in quantifying the different facets of sexual behaviour and complexity of sexual networks.⁵

The theoretical underpinnings of sexual partnership dynamics have received much attention in the last two decades.^{2–6–8} An underlying philosophy of this line of investigation was to identify the

plausible and testable stochastic processes that can explain the observed patterns of sexual partnering.⁶ The strengths of this research were in elucidating causal mechanisms that can generate the macrobehaviour of individual actors and in potentially furnishing methodologies for estimating measures of interest to inform practical applications.⁶

Building on this progress, we describe here a stochastic process model for understanding the dynamics of non-cohabiting heterosexual sex partnering. The focus of our approach however is not theoretical, but pragmatic: our immediate aim is to use existing sexual behaviour data to generate inferences about structure of sexual networks and to map patterns of non-cohabiting sex across sub-Saharan Africa (SSA), the region most affected by the HIV epidemic. Accordingly, we present an estimation methodology for characterising non-cohabiting sex partnering and apply it to 25 countries in SSA using nationally representative data, that of the Demographic and Health Surveys (DHS).⁹

Since the majority of HIV incidence in SSA is estimated to arise outside the context of marital or cohabiting partnerships,^{10–11} our study contributes to improved understanding of HIV epidemiology in this continent. More broadly, this empirically driven understanding of non-cohabiting sex has the potential to empower future epidemiological analyses at the heart of the intersection between population sexual behaviour and STI epidemiology. Such analyses may use different methodological approaches, among them statistical analysis and mathematical modelling, and may address a variety of open scientific questions.

METHODS

Conceptual framework and mathematical model

We assumed that the formation or dissolution of a non-cohabiting sex partnership follows a Poisson stochastic process. Specifically, we assumed that there is a fixed hazard per unit time to form a partnership. If a partnership is formed, there is also a different fixed hazard per unit time for this partnership to be dissolved. Therefore, the equilibrium distribution of the number of non-cohabiting partners for an individual in the population (individual 'x') is described by

$$F_x \equiv \text{Poisson} \left(\frac{p_x}{\mu_x} \right). \quad (1)$$

Here, Poisson denotes the Poisson distribution, p_x denotes the probability of a partner acquisition for

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individual 'x' per unit time and μ_x denotes that for partnership dissolution. The equilibrium here is a dynamic equilibrium of the underlying behavioural process,^{6 12} the equilibrium solution of the Kolmogorov forward equation for the stochastic process (Derivation S1 in the online supplementary appendix).

Figure 1A illustrates a number of non-cohabiting partnerships recorded by a cross-sectional survey, such as that of the DHS, at some time $t_1=t_0+T$. Here, T is the survey's target reporting period, normally 12 months in the DHS, where participants are asked about the number of non-cohabiting partners they have had over the last 12 months. Each participant would report his/her total number of non-cohabiting partners during T, that is, between the beginning of the survey's target period at t_0 and the time of the actual survey at t_1 . The total number of reported partners for each individual is given by the sum of the number of partners at t_0 (denoted by white circles in figure 1A and described by the distribution F_x) and the number of newly formed partners during T (denoted by black circles in figure 1A). The latter is described by the distribution:

$$H_x \equiv \text{Poisson}(p_x T). \quad (2)$$

Accordingly, the distribution of the total number of partners over T for individual 'x' is given by:

$$D_x \equiv \text{Poisson}\left(p_x \left(\frac{1}{\mu_x} + T\right)\right), \quad (3)$$

and the expected value of the total number of partners is given by:

$$E_x = p_x \left(\frac{1}{\mu_x} + T\right). \quad (4)$$

Human sexual behaviour is marked by heterogeneity. Informed by empirical data and previous theoretical work,^{2 3 6 8} and to accommodate wider flexibility,⁶ we assumed that the population distribution of the individual mean values of the number of partners follows a gamma distribution with k and θ parameters:

$$Z_p \equiv \text{Gamma}(k, \theta). \quad (5)$$

The parameter k determines the shape of the gamma distribution with different values generating a variety of shapes. The parameter θ scales the distribution.

Based on the above description, the distribution of the reported number of partners in a cross-sectional survey is given by:

$$Q_s \equiv \text{Poisson}(E_x \sim \text{Gamma}(k, \theta)) = \text{NB}(k, \frac{1}{1+\theta}), \quad (6)$$

where $\text{NB}(k, 1/1 + \theta)$ denotes the negative-binomial distribution parameterised by k and $1/(1+\theta)$. $\text{NB}(k, 1/1 + \theta)$ provides the distribution of the number of failures until k successes in Bernoulli trials where the success probability is $1/(1+\theta)$. The theoretical links between all of these distributions are illustrated in figure 1B.

Estimation of distribution parameters

According to the above analysis, it is possible to characterise non-cohabiting sex partnership formation and dissolution in a population using only two parameters: k and θ ; the shape and scale parameters of the Z_p distribution. We estimated these parameters stratified by marital status and sex for 25 countries in SSA using DHS data. We also calculated, through these

parameters, the country-specific mean values ($k\theta$) and variances ($k\theta^2$) of the number of partners over the last 12 months.

k and θ were estimated using a maximum likelihood method of the function:

$$L(k, \theta) = \prod_x \text{pmf}\left(\text{NB}(k, \frac{1}{1+\theta}), x\right). \quad (7)$$

Here, $\text{pmf}(\text{NB}(k, 1/1 + \theta), x)$ denotes the probability mass function of the Q_s distribution conditioned on the observed outcome of the number of partners for individual 'x'. The maximum likelihood estimation was implemented in MATLAB¹³ using the *nbinfit* function. In occasions when the empirical mean was larger than that of variance, the negative-binomial function was replaced by its limit as a Poisson function, and the maximum likelihood estimation was performed using the *poissfit* function. The 95% CIs for the mean values and variances were calculated by bootstrap resampling.

Model fitting

The model was fitted using data from the most recent DHS round for all countries with DHS data in SSA. DHS are nationally representative household surveys that collect individual-level demographic and health data.⁹ We analysed a total of 25 countries: Burkina Faso (2010), Burundi (2010), Cameroon (2011), Democratic Republic of Congo (2007), Congo-Brazzaville (2009), Cote d'Ivoire (2012), Ethiopia (2011), Ghana (2003), Guinea (2005), Kenya (2008–2009), Lesotho (2009), Liberia (2007), Malawi (2010), Mali (2006), Mozambique (2009), Niger (2006), Rwanda (2010), Sao Tome and Principe (2008–2009), Senegal (2010–2011), Sierra Leone (2008), Swaziland (2006–2007), Tanzania (2012), Uganda (2011), Zambia (2007) and Zimbabwe (2011).

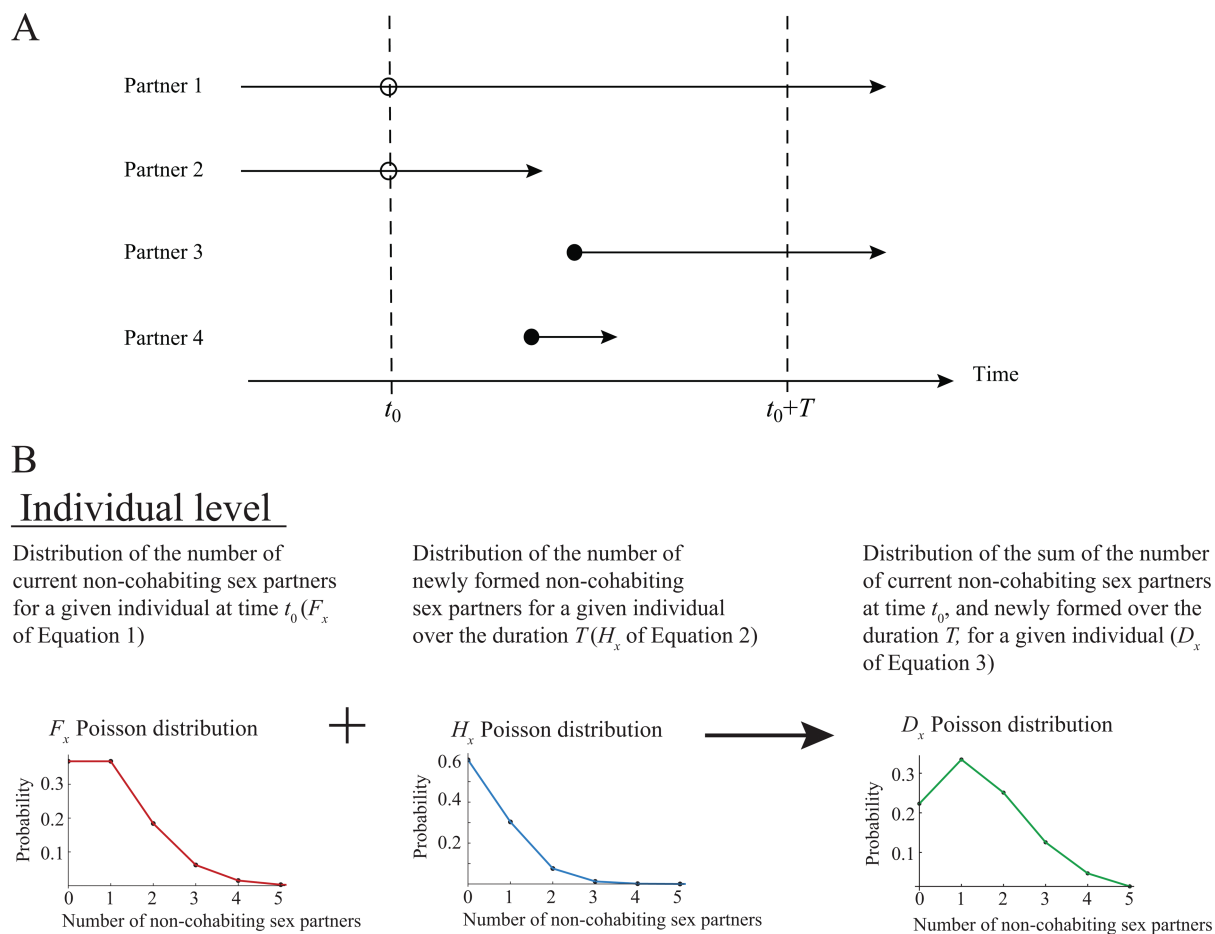
We defined a non-cohabiting sexual partnership as any reported sexual encounter between a man and a woman outside marriage or cohabitation. For each country, we extracted the empirical distribution of the number of non-cohabiting partners over the last 12 months stratified by marital status (married/unmarried) and sex (male/female). Descriptive statistics for these distributions can be found in online supplementary table S1.

RESULTS

For the majority of countries and subpopulations, our model-predicted distributions matched the empirical DHS distributions (figure 2A and online supplementary figures S1–S4). For few countries, however, the number of non-cohabiting partners reported by unmarried men and women showed a peak in frequency at one (ie, when a single partner was reported). This peak at one was not captured by the model in these countries, although overall the predicted distributions still matched well the empirical distributions (figure 2B and online supplementary figures S2 and S4).

There was heterogeneity with respect to marital status and sex in the model-estimated mean values for number of partners and associated 95% CIs. Unmarried men and women showed larger mean values and wider 95% CIs than their married counterparts (figure 3). Men showed larger mean values and wider 95% CIs than women (figure 3). The median across all country-specific mean values was highest for unmarried men at 0.574 partners over the last 12 months, followed by that of unmarried women at 0.337 and then that of married men at 0.192. Married women had the lowest median across SSA at 0.038 partners.

The estimated mean values varied also across countries. The largest variability was among unmarried men ranging from 0.103 to 1.206 partners. The range for unmarried women was



Population level

Distribution of the sum of the number of current non-cohabiting sex partners at time t_0 , and newly formed over the duration T , for a given individual (D_x of Equation 3)

Distribution of the “tendency” to form non-cohabiting sex partnerships in the population (Z_p of Equation 5)

Distribution of the reported number of non-cohabiting sex partners over the 12 months in the cross-sectional survey of this population at time t_0+T (Q_s of Equation 6)

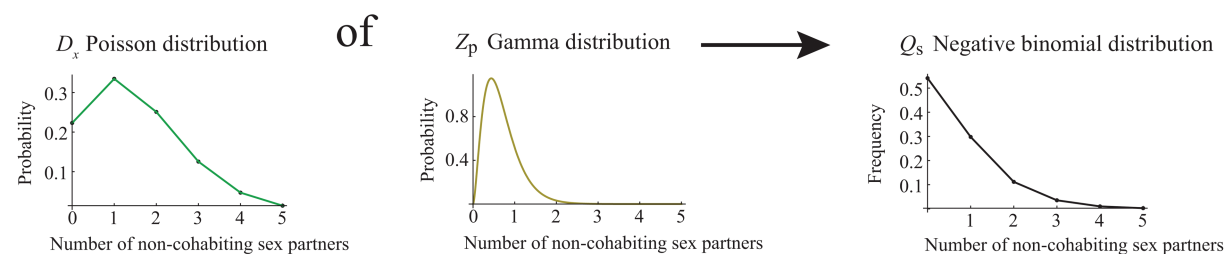


Figure 1 Illustration of how a cross-sectional survey can assess the number of non-cohabiting sex partners during a specified duration ‘ T ’. (A) Schematic diagram illustrating the process of non-cohabiting sex partnership formation for an individual in a population. The white circles indicate the number of partners at the beginning of the survey’s target period ‘ t_0 ’. The black circles indicate the number of newly acquired partners over the survey’s target period ‘ T ’ following t_0 . The survey asks participants about the number of non-cohabiting sex partners over T (normally the last 12 months). Each arrow indicates the partnership duration. (B) A schematic diagram of the conceptual framework for the stochastic process model used to characterise the distribution of the reported number of non-cohabiting sex partners over the last 12 months.

0.059 to 0.810. The ranges for married men and women were 0.009 to 0.549 and 0.003 to 0.098, respectively.

The model-estimated variances also exhibited heterogeneity with respect to marital status and sex. Men showed overall larger variances and wider 95% CIs than women (figure 4). Unmarried men showed overall larger variances than married men. Unmarried and married women showed very small variances. The median across all country-specific variances was

highest for unmarried men at 0.127 followed by married men at 0.057. The medians for unmarried and married women were 0.000 and 0.003, respectively (figure 4).

The model-estimated variances also varied across countries. The largest variability was observed among unmarried women (ranging from 0.000 to 1.994), followed by unmarried men (0.000 to 1.580), married men (0.002 to 0.908), and lastly married women (0.000 to 0.153).

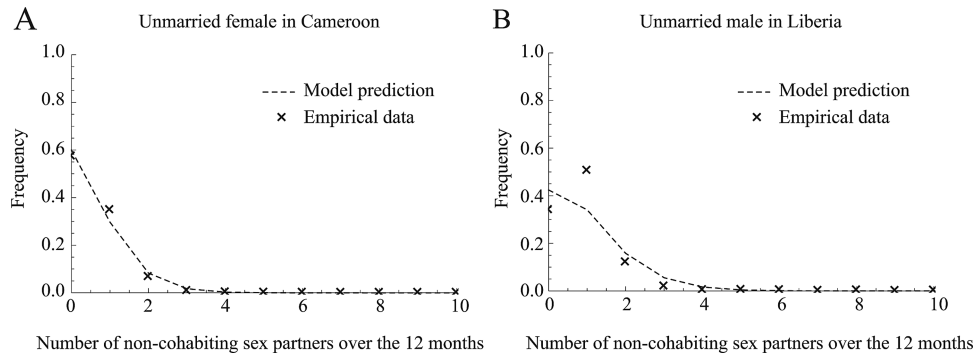


Figure 2 Illustration of the model fits of empirical distributions. (A) An example of a robust model fit of the number of non-cohabiting sex partners over the last 12 months. Robust fits were found for the majority of countries. (B) An example of a non-optimal model fit of the number of non-cohabiting sex partners over the last 12 months. Less than optimal fits were found for only unmarried men and women in few countries. All fits stratified by marital status and sex in the 25 studied countries in sub-Saharan Africa can be found in online supplementary figures S1–S4.

The model-estimated coefficients of variation (CV) also varied across countries. The largest variability was observed among married women (ranging from 0.000 to 20.324), followed by

unmarried women (0.000 to 7.415), married men (0.555 to 5.845), and lastly unmarried men (0.000 to 2.857). Among the 25 countries, the model-estimated CV was equal to zero in zero

Figure 3 Estimated mean values and associated 95% CIs of the number of non-cohabiting sex partners over the last 12 months stratified by marital status and sex in 25 countries in sub-Saharan Africa. Detailed information on the empirical measures can be found in online supplementary table S1.

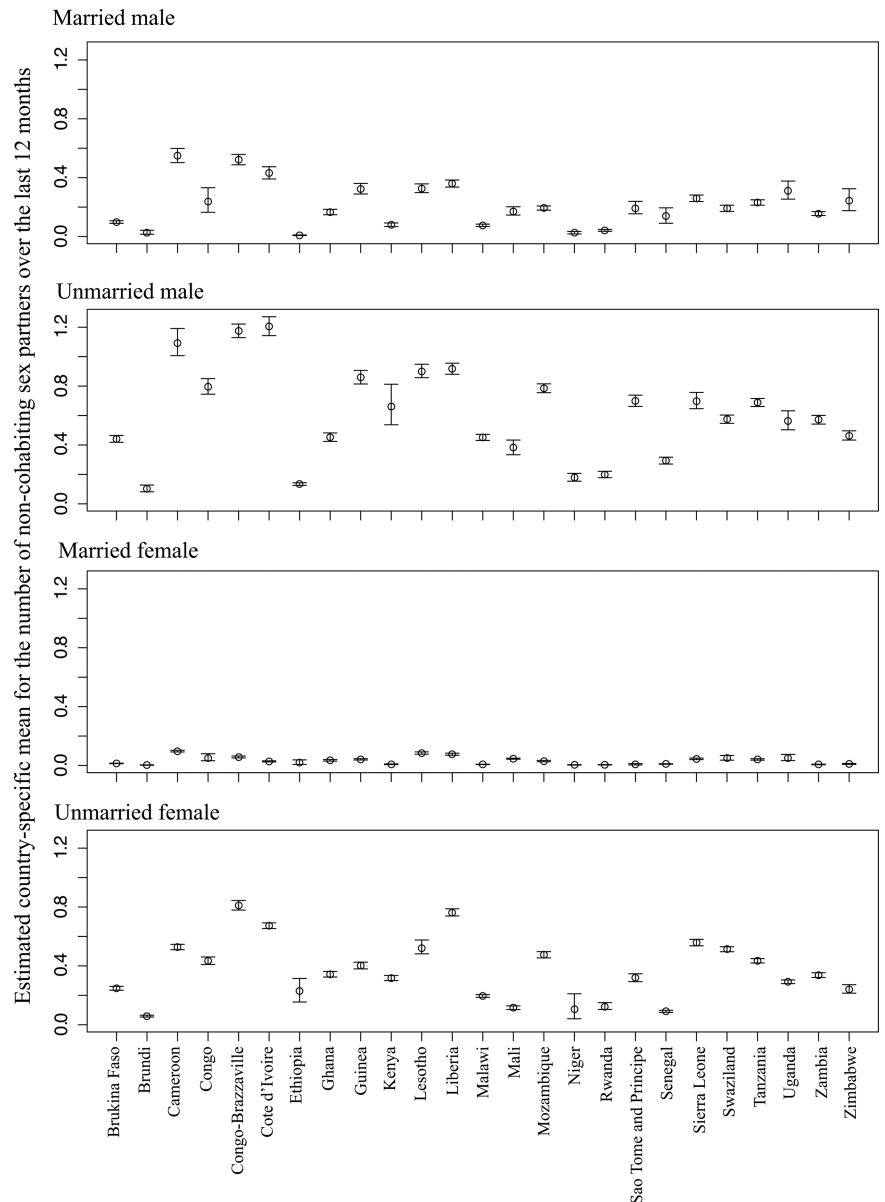
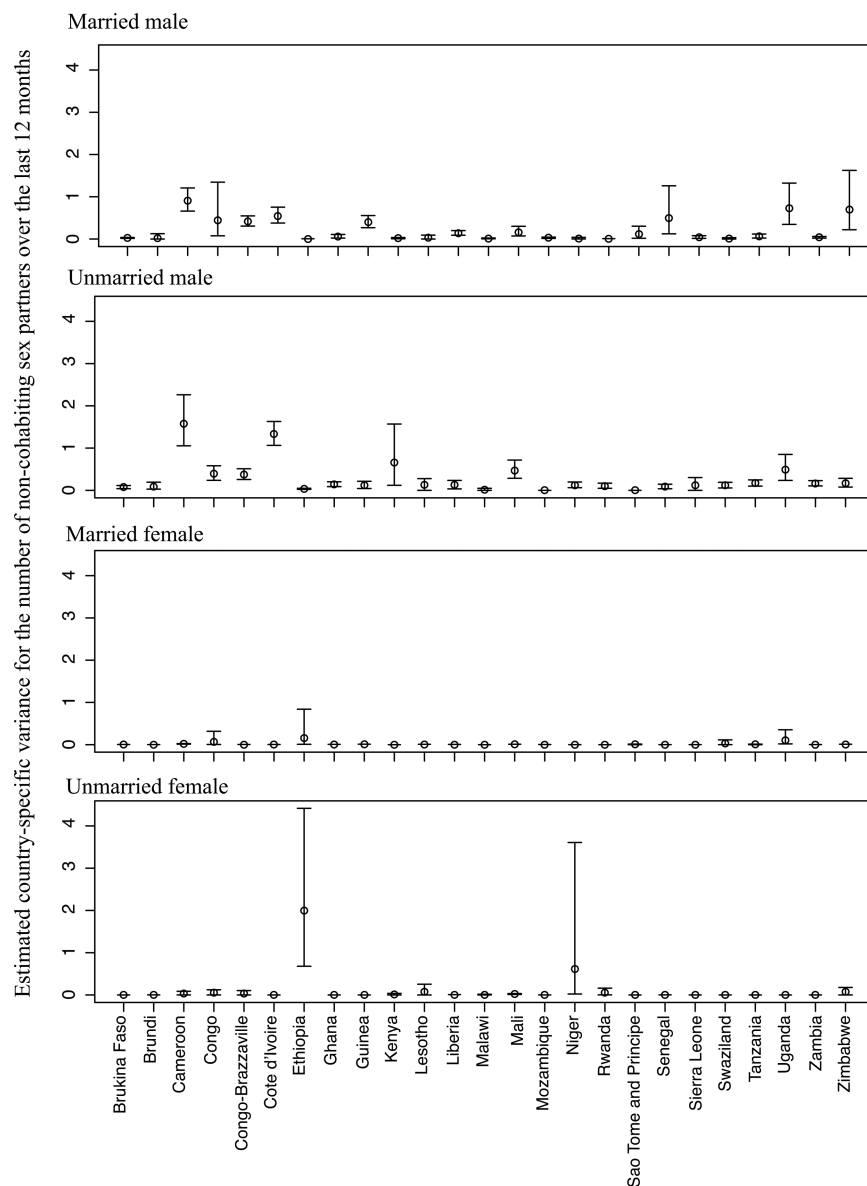


Figure 4 Estimated variances and associated 95% CIs of the number of non-cohabiting sex partners over the last 12 months stratified by marital status and sex in 25 countries in sub-Saharan Africa. Detailed information on the empirical measures can be found in online supplementary table S1.



country for married men, two countries for unmarried men, four countries for married women, and 14 countries for unmarried women.

DISCUSSION

We described an analytical framework for understanding and characterising the process of non-cohabiting sex partnership formation and dissolution. We applied this methodology to 25 countries in SSA to derive the distribution of the number of sex partners over the last 12 months and to estimate summary statistics for each of married and unmarried men and women. Accordingly, we provided an overall mapping of the patterns of non-cohabiting sex partnering across much of SSA.

Our theoretical approach was expressed in terms of a parsimonious stochastic process model that included only two fitting parameters. The model-predicted distributions fitted the empirical distributions for the majority of countries. The agreement between the predicted and empirical distributions was remarkable in all four studied strata including married and unmarried men and married and unmarried women.

The ability of this model to reproduce the empirical distributions suggests that at least the gross features of non-cohabiting

sex networking, which is believed to be a complex phenomenon,⁸ can be understood in terms of few rules dictating a simple and identifiable stochastic process. Our findings therefore add an insight to our understanding of premarital and extramarital sexuality.

These results suggest that there is a propensity to acquire non-cohabiting sex partners for any individual in a population, but the strength of this propensity varies from one individual to another. It seems that non-cohabiting sex is a random 'opportunistic' phenomenon whose expression is constrained by the circumstances of each individual. Past does not appear to be a circumstantial factor (Poisson process), but the social context of the individual, along with personal attributes and beliefs system, matter. The large heterogeneity in individual contexts in a society creates a distribution of 'opportunities' to engage in non-cohabiting sex, and this distribution appears to follow the shape of that of a gamma distribution.

Since human societies generally regard non-cohabiting sex as socially undesirable, this limits the latitude for engagement in sex outside sanctioned marriage. When an opportunity arises for non-cohabiting sex in the absence of serious perceived negative consequences, non-cohabiting sex may occur. If this

interpretation is valid, men should engage more in non-cohabiting sex than women, since female sexuality is globally more socially constrained, and unmarried individuals should engage in non-cohabiting sex more so than married individuals. Just as there is a distribution of 'opportunities' within any society, and given the diversity of human societies, there should be also variability across societies in the 'mean opportunity' to engage in non-cohabiting sex.

This interpretation is consistent with the results of our analyses. The model-estimated mean values and variances of the number of non-cohabiting partners suggest wide variation by country, sex and marital status. The mean values across countries varied by as much as an order of magnitude, and men had larger variances than women. Unmarried men and women had much larger mean values of partners than their married counterparts. While married men still reported considerable non-cohabiting sex, this was not the case for married women.

Other evidence appears also to support such understanding of non-cohabiting sex. Non-cohabiting sex is associated empirically with 'possibility factors',¹⁴ such as time spent apart in a spousal or cohabiting partnership (eg, through occupational travel), less reliance of women on men for their livelihood or living in higher population densities.¹⁴⁻¹⁶ Our findings are also in agreement with previous studies examining the statistical properties of different sexual partnership distributions. These studies have shown that non-homogenous Poisson models, just as the one described here, produce optimal fits of empirical data.^{6 8 17}

Our results suggest a disproportionate role for unmarried individuals in driving heterogeneity in sexual networks, at least in SSA. This is probably not surprising considering that close to half of HIV incidence in SSA occurs among young adults, possibly through non-cohabiting sex.^{18 19} However, the small mean values and limited variances for women, especially those married, do not seem to be compatible with the comparable HIV prevalence among men and women,⁹ and the nearly equal probability for both sexes to be the index partner in an HIV serodiscordant couple in SSA.^{20 21} This may suggest under-reporting of non-cohabiting sex or participation bias among women. This suggestion is plausible considering the challenges of sexual-behaviour data collection,⁵ gender differentials in reporting of sexual behaviour²² and biomarker studies showing under-reporting of recent unprotected intercourse by women.^{23 24} There is also evidence that sexual behaviour surveys may not be capturing high sexual risk women such as commercial sex workers.^{22 25}

Furthermore, both the estimated mean values and variances of non-cohabiting sex in all strata seem lower, in light of global measures,²⁶ than what would be expected in a context of such high HIV prevalence in SSA.⁹ Mathematical modelling suggests that high variance in sexual behaviour is essential to explain the size of the HIV epidemics seen in SSA.^{3 8} This further suggests reporting or participation bias in the surveys which may have, along with censorship of large number of sexual partners, altered the tail of the empirical distributions for the number of partners. This also possibly explains the outlier variances seen in few countries (figure 4). Such limitations in self-reported data may influence the explanatory power of sexual behaviour analyses including those presented here. The availability of detailed and objective sexual behaviour data in the future, such as with the addition of biomarkers,²⁷ may facilitate a more refined and in-depth understanding of non-cohabiting sex.

For few countries, the model did not yield optimal fits to the empirical distributions for unmarried men and women, as it failed to capture a peak in frequency at one (see online

supplementary figures S2 and S4). The DHS question that enquires about non-cohabiting sex does not distinguish between long-term and short-term non-cohabiting partnerships.⁹ This peak at one may reflect a tendency among unmarried individuals in a few countries to engage in a single long-term non-cohabiting partnership. Potential ambiguity in the definition of non-cohabiting sex for some individuals may also contribute to explaining this peak at one.

The overall excellent agreement between model predictions and empirical data cannot exclude the possibility that other stochastic process models, with varying assumptions, may fit equally well the empirical distributions. It has been shown that sexual partnership distributions can be described using different stochastic process models,^{6 8} and that there may not be a unitary process underlying the formation of sexual networks.⁶ For example, it is conceivable that there could be penalties for acquiring multiple concurrent partners, and therefore the Poisson assumption may not be a realistic assumption with the addition of more partners. With only the gross features of sexual behaviour being captured in surveys, not to mention the known non-random biases in self-reported data,⁵ it is challenging to have a fine-grained understanding of the diverse human sexual networks.

Notably, capturing the tail of partner distributions, which disproportionately influences STI epidemiology,²⁸⁻³⁰ continues to be a difficult challenge.⁶ The tail plays a critical role in determining the variance, and thereby heterogeneity in sexual networks, but the information content at the tail is limited with the small number of participants reporting large number of partners even in large surveys.⁶ This challenge can be seen in the variability of the size of the variance CIs and in the variability of the model-estimated and survey variances across countries (figure 4 and online supplementary table S1). Nevertheless, our model appears to provide a satisfactory degree of precision and a practical description of non-cohabiting sex dynamics.

In conclusion, we described an analytical framework in terms of a parsimonious stochastic process model to characterise non-cohabiting sex partnering in SSA. The model-predicted distributions fitted nicely the empirical distributions for the majority of countries. The estimated mean values and variances of the number of non-cohabiting partners suggest wide variation by country, sex and marital status. Unmarried individuals, particularly unmarried men, appear to play a major role in driving heterogeneity in sexual networks. Unmarried men and women had much larger mean values of number of partners than their

Key messages

- ▶ Non-cohabiting sex partnering in human populations appears to be a random 'opportunistic' phenomenon and can be described by a simple stochastic process.
- ▶ Mean values and variances of the number of non-cohabiting sex partners in sub-Saharan Africa show wide variation by country, sex and marital status.
- ▶ Mean values and variances of sex partners, especially for women, appear to be smaller than what is expected in a context of large HIV epidemics.
- ▶ Unmarried men appear to play a disproportionate role in driving heterogeneity in sexual networks, and possibly epidemiology of sexually transmitted infections, in sub-Saharan Africa.

married counterparts. While married men still reported considerable non-cohabiting sex, this was not the case for married women. These findings add fresh insights to our understanding of premarital and extramarital sexuality and have the potential to empower further statistical and mathematical modelling analyses at the intersection between population sexual behaviour and STI epidemiology.

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Contributors RO conceived the study, developed the mathematical model and performed the analyses. HC contributed to the data analyses. LJA-R led the conception and conduct of the study. All authors contributed to the interpretation of the results and drafting of the manuscript.

Competing interests None.

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目的：サブサハラアフリカ地域における婚姻関係を結んでいない性交渉相手とのパートナーシップ形成過程を、国単位での性行動調査データを用い明らかにする。

方法：婚姻関係を結んでいない性交渉相手とのパートナーシップ形成過程は非定常ポアソン過程で現せるとして数理モデルを構築した。構築された数理モデルをサブサハラアフリカ25カ国の性行動調査データに適用し、国別での12ヶ月間で新しく誕生した非婚姻関係の性交渉パートナーシップ数の平均と分散を推定した。

結果：我々の提案したモデルは未婚者、既婚者、性別を問わず、性行動データと良くあてはまっていた。新しく形成された国別平均パートナーシップ数の25カ国間における中央値は、0.574と未婚男性が最も高く、未婚女性(0.337)、既婚男性(0.192)、既婚女性(0.038)と続いた。国別パートナーシップ数の分散に関しては未婚男性が最も高く(0.127)、既婚男性(0.057)、未婚女性(0.003)、既婚女性(0.000)と続いた。国の違いによる平均パートナーシップ数のばらつきは未婚男性が最も大きく(0.103から1.206)、分散のばらつきは未婚女性が最も大きかった(0.000から1.994)。

結論：非婚姻関係の性交渉パートナーシップは偶然の効果に大きく左右される確率論的な現象である。また、サブサハラアフリカ地域では、非婚姻関係の性交渉パートナーシップ数の平均、分散共に、国、当事者の婚姻状況、性別によって大きく異なり、未婚者は既婚者よりも平均パートナー数が大きく、また男性の方が女性よりも平均パートナー数が大きい。未婚者の性行動が性的関係ネットワーク、ひいては性感染症の流行を決定づける大きな役割を果たしている事が示唆される。

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Supplementary Online Appendix

Dynamics of non-cohabiting sex partnering in sub-Saharan Africa

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Derivation S1

We assumed that the formation or dissolution of a non-cohabiting sex partnership follows a Poisson stochastic process. We assumed that there is a fixed hazard per unit time to form a partnership, p_x . Moreover, if a partnership is formed, there is a fixed hazard per unit time for this partnership to be dissolved, μ_x .

The equilibrium distribution for the number of partners for individual x , F_x , is obtained using the Kolmogorov forward equation for this process. The probability that the individual x has m partners at a specific time $t + \Delta t$, $\Pr(n_x = m, t + \Delta t)$, is described in terms of the probabilities that this individual has $m - 1$ ($\Pr(n_x = m - 1, t)$), m ($\Pr(n_x = m, t)$), and $m + 1$ ($\Pr(n_x = m + 1, t)$) partners at t . It follows then, to first order in Δt , that

$$\begin{aligned}\Pr(n_x = m, t + \Delta t) &= (1 - m\mu_x\Delta t - p_x\Delta t)\Pr(n_x = m, t) \\ &\quad + p_x\Delta t\Pr(n_x = m - 1, t) \\ &\quad + (m + 1)\mu_x\Delta t\Pr(n_x = m + 1, t)\end{aligned}$$

$$\begin{aligned}\Pr(n_x = 0, t + \Delta t) &= (1 - p_x\Delta t)\Pr(n_x = 0, t) \\ &\quad + \mu_x\Delta t\Pr(n_x = 1, t)\end{aligned}$$

As $\Delta t \rightarrow 0$, we obtain the Kolmogorov differential equation:

$$\begin{aligned}\frac{d\Pr(n_x = m, t)}{dt} &= -(p_x + m\mu_x)\Pr(n_x = m, t) + p_x\Pr(n_x = m - 1, t) + (m + 1)\mu_x\Pr(n_x = m + 1, t) \\ \frac{d\Pr(n_x = 0, t)}{dt} &= -p_x\Pr(n_x = 0, t) + \mu_x\Pr(n_x = 1, t)\end{aligned}$$

Accordingly, the probability for having m partners at equilibrium is given by

$$\Pr(n_x = m)^* = \frac{\exp\left[-\frac{p_x}{\mu_x}\right]\left(\frac{p_x}{\mu_x}\right)^m}{m!}.$$

This probability mass function is equivalent to the probability mass function of a Poisson distribution with a mean of $\frac{p_x}{\mu_x}$. Therefore, the equilibrium distribution for the number of non-cohabiting sex partners for individual x is described by

$$F_x \equiv \text{Poisson}\left(\frac{p_x}{\mu_x}\right).$$

Table S1: Descriptive statistics for the reported number of non-cohabiting sex partners in the last 12 months across 25 countries in sub-Saharan Africa.

Country	Year	Males						Females					
		Married			Unmarried			Married			Unmarried		
		<i>n</i>	Mean	Variance	<i>n</i>	Mean	Variance	<i>n</i>	Mean	Variance	<i>n</i>	Mean	Variance
Burkina Faso	2010	4536	0.09830	0.1310	2761	0.441	0.562	13390	0.01410	0.01680	3694	0.2480	0.2330
Burundi	2010	2423	0.02680	0.1680	1855	0.103	0.365	5259	0.00323	0.00360	4126	0.0589	0.0588
Cameroon	2011	3618	0.54900	3.0300	3549	1.090	11.40	9792	0.09760	0.12200	5614	0.5280	0.7120
Congo	2007	2757	0.23700	6.9400	1971	0.796	2.040	6560	0.05050	1.48000	3400	0.4340	0.8280
Congo-Brazzaville	2009	3062	0.52200	1.4300	2785	1.180	2.160	3943	0.05810	0.05980	2596	0.8100	0.9970
Cote d'Ivoire	2012	2679	0.43100	1.6900	2446	1.210	3.740	6442	0.02790	0.03030	3597	0.6730	0.5050
Ethiopia	2011	7909	0.00872	0.0132	6178	0.134	0.183	10188	0.01920	0.94600	6303	0.2300	14.500
Ghana	2003	2724	0.16600	0.3450	2286	0.453	0.712	3691	0.03500	0.03860	1997	0.3440	0.2730
Guinea	2005	1877	0.32400	0.9500	1287	0.860	1.040	6313	0.04180	0.04990	1618	0.4020	0.3260
Kenya	2008-09	1818	0.07970	0.1050	1639	0.662	11.90	5028	0.00875	0.00868	3395	0.3170	0.3840
Lesotho	2009	1378	0.32700	0.4380	1885	0.901	1.430	4099	0.08470	0.08920	3435	0.5210	2.9500
Liberia	2007	3274	0.36000	0.6950	2653	0.917	1.390	4367	0.07650	0.07840	2532	0.7630	0.5510
Malawi	2010	4196	0.07600	0.1020	2960	0.452	0.485	15423	0.00772	0.00766	7563	0.1960	0.2260
Mali	2006	2744	0.17100	0.8090	1430	0.382	1.350	12301	0.04610	0.05320	2257	0.1160	0.1350
Mozambique	2009	3084	0.19300	0.2310	1696	0.785	0.574	4251	0.02990	0.03180	2160	0.4760	0.3750
Niger	2006	2115	0.02510	0.0538	1409	0.179	0.365	7413	0.00418	0.00444	1786	0.1060	5.2600
Rwanda	2010	3260	0.04140	0.0465	3068	0.199	0.545	6832	0.00542	0.00539	6835	0.1230	1.4400
Sao Tome and Principe	2008-09	1177	0.19200	0.7860	1101	0.699	0.592	1740	0.00805	0.01260	859	0.3200	0.2460
Senegal	2010-11	2158	0.13900	2.2200	2770	0.293	0.547	10804	0.01030	0.01050	4884	0.0913	0.0912
Sierra Leone	2008	1895	0.26000	0.3400	1215	0.698	1.390	5209	0.04530	0.04440	1977	0.5580	0.3560
Swaziland	2006-07	1265	0.19100	0.2100	2870	0.574	0.839	2067	0.05030	0.21800	2917	0.5130	0.2970
Tanzania	2012	4329	0.23000	0.5410	4005	0.688	1.140	6816	0.04090	0.05830	4127	0.4340	0.3390
Uganda	2011	5712	0.31200	7.8600	3873	0.562	6.020	7626	0.05060	1.31000	4523	0.2920	0.2550
Zambia	2007	3618	0.15600	0.2230	2864	0.571	0.914	4314	0.00719	0.00760	2828	0.3370	0.2650
Zimbabwe	2011	3747	0.24400	7.8100	3635	0.462	1.340	5518	0.01090	0.01870	3537	0.2410	1.1300

**n* denotes the sample size; mean and variance denote the mean and variance of the reported number of non-cohabiting sex partners in the last 12 months as reported in the cross-sectional Demographic and Health Survey.

Figure S1: Comparison of model-predicted distributions and empirical distributions for married males. Distribution of the number of non-cohabiting sex partners over the last 12 months among married males across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.

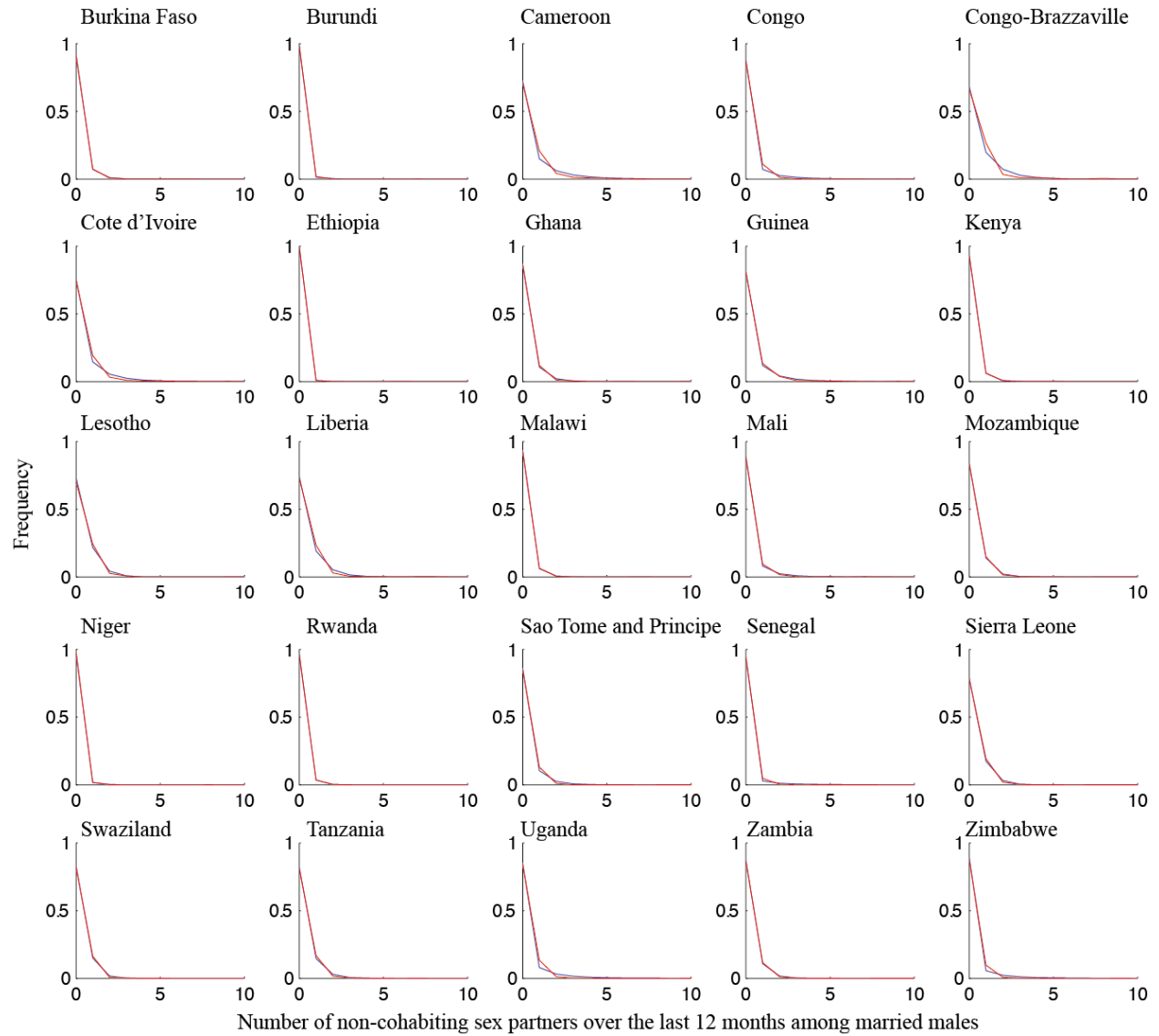
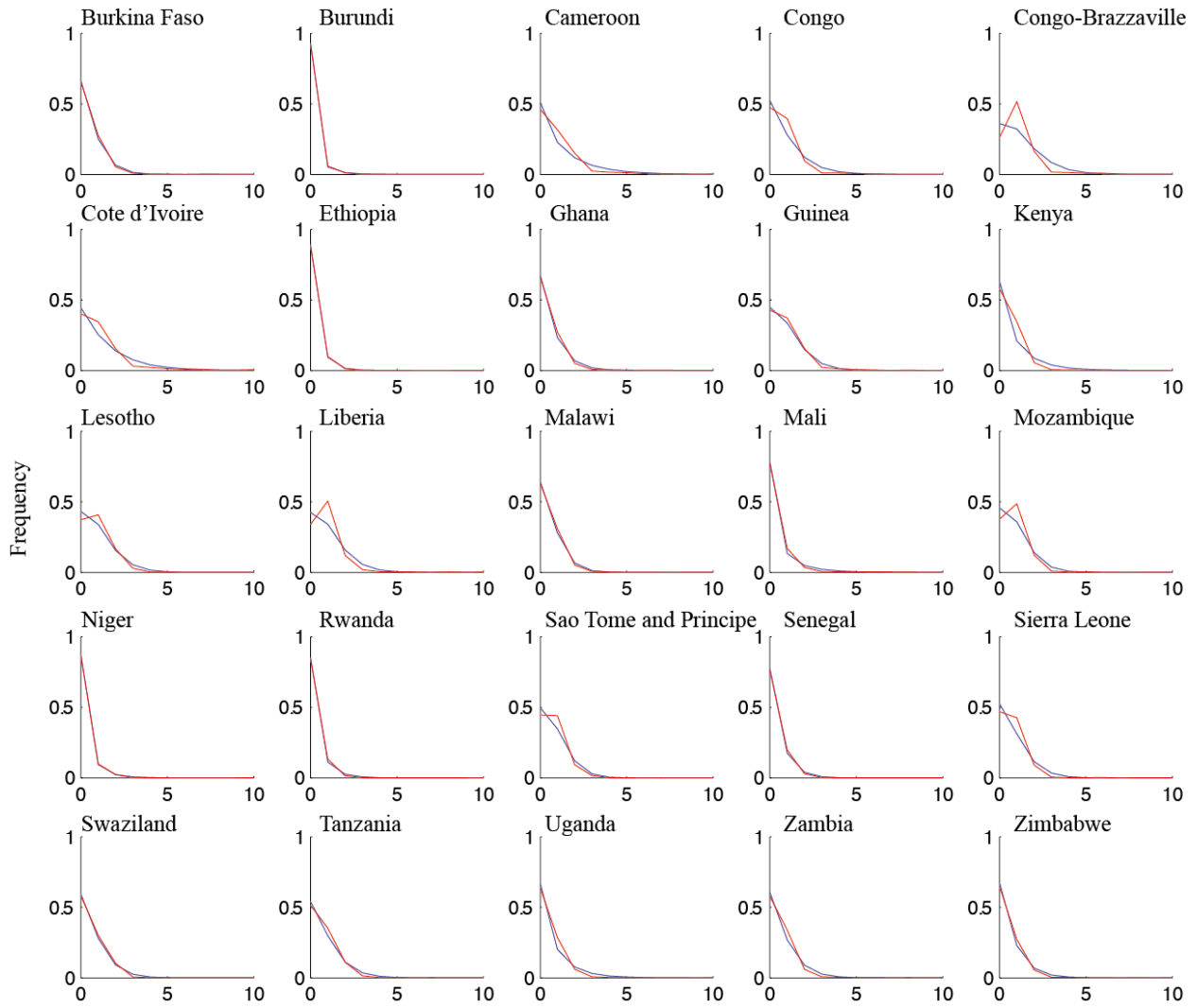


Figure S2: Comparison of model-predicted distributions and empirical distributions for unmarried males. Distribution of the number of non-cohabiting sex partners over the last 12 months among unmarried males across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.



Number of non-cohabiting sex partners over the last 12 months among unmarried males

Figure S3: Comparison of model-predicted distributions and empirical distributions for married females. Distribution of the number of non-cohabiting sex partners over the last 12 months among married females across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.

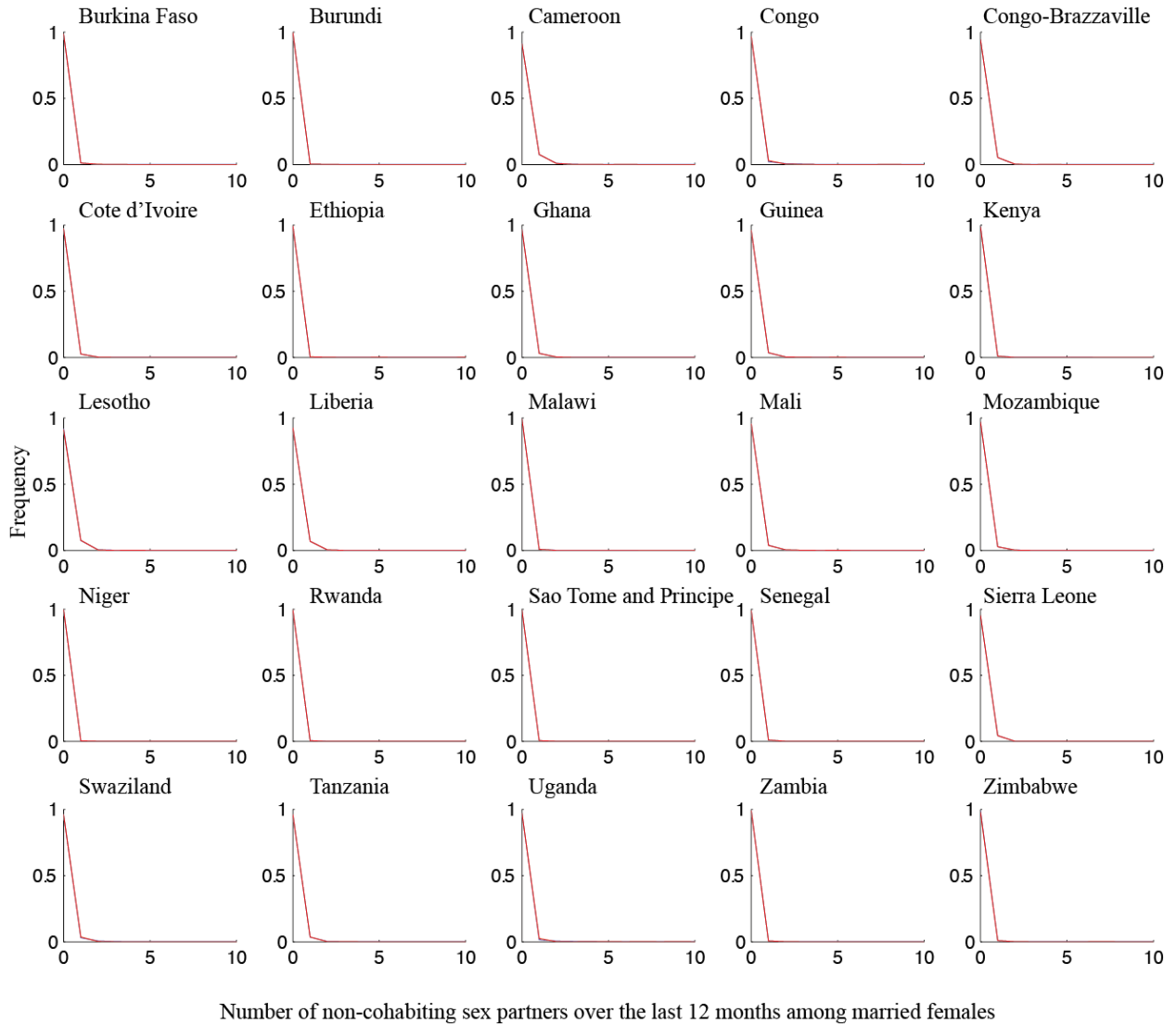


Figure S4: Comparison of model-predicted distributions and empirical distributions for unmarried females. Distribution of the number of non-cohabiting sex partners over the last 12 months among unmarried females across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.

